

Generating Hermite polynomial excited squeezed states by means of conditional measurements on a beam splitter

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A scheme for conditional generating a Hermite polynomial excited squeezed vacuum states (HESVS) is proposed. Injecting a two-mode squeezed vacuum state (TMSVS) into a beam splitter (BS) and counting the photons in one of the output channels, the conditional state in the other output channel is just a HESVS. To exhibit a number of nonclassical effects and non-Gaussianity, we mainly investigate the photon number distribution, sub-Poissonian distribution, quadrature component distribution, and quasi-probability distribution of the HPESVS. We find that its nonclassicality closely relates to the control parameter of the BS, the squeezed parameter of the TMSVS, and the photon number of conditional measurement. These further demonstrate that performing the conditional measurement on a BS is an effective approach to generate non-Gaussian state.

ocis: (270.5570) Quantum detectors; (270.4180) Multiphoton processes; (270.5290) Photon statistics

Keywords: Conditional measurement; beam splitter; Wigner function; Nonclassicality

I. INTRODUCTION

Quantum state engineering has been a subject of increasing interest to construct various novel nonclassical states in quantum optics and quantum information processing[1, 2]. From a theoretical point of view, the simplest way of generating nonclassical field states is to apply the photon creation operation to classical states such as the thermal and coherent states[3–5]. These nonclassical states, such as the single-photon added coherent state[6] and single-photon-added thermal state[7], have been realized experimentally. Subsequently, it has been demonstrated that subtracting photons from traditional quantum states exhibit an abundance of nonclassical properties[8–11]. Photon subtraction or addition can improve entanglement between Gaussian states[12], loophole-free tests of Bell's inequality[13], and quantum computing[14].

To meet the requirement of the development of quantum optics and quantum information tasks, some nonclassical states are explored by performing the different combination of photon subtraction and photon addition[15–20], which have different properties. Kim et al[21] discussed single photon adding then subtracting (or single photon subtracting then adding) coherent state (or thermal state) to probe quantum commutation rules $[a, a^\dagger] = 1$. Lee et al[17] investigated the nonclassicality of field states when photon subtraction-then-addition operation or the photon addition-then-subtraction operation is applied to the coherent state (or thermal state), respectively. Yang and Li[18] analyzed multiphoton addition followed by multiphoton subtraction ($a^l a^{\dagger k}$) and its inverse ($a^{\dagger l} a^k$) on an arbitrary state. Recently, Lee and

Nha[23] proposed a coherent superposition of photon addition and subtraction, $ta + ra^\dagger$ ($|t|^2 + |r|^2 = 1$) acting on a coherent state and a thermal state. More recently, we investigated the nonclassical properties of optical fields generated by Hermite-excited coherent state[24] and Hermite-excited squeezed thermal states[25].

On the other hand, another promising method for generating highly nonclassical states of optical fields is known to be conditional measurement[26–30]. Namely, when a system is prepared in an entangled state of two subsystems and a measurement is performed on one subsystem, then the quantum state of the other subsystem can be reduced to a new state. In particular, it turned out that conditional measurement on a beam splitter may be advantageously used for generating new classes of quantum states[29, 30]. Dakna's group used conditional measurement on the BS to generate cat-like state[31]. Podoshvedov et al[28] proposed optical scheme for generating both a displaced photon and a displaced qubit via conditional measurement. In Ref.[32], they proposed to create arbitrary Fock states via conditional measurement on the BS. In addition, conditional output measurement on the BS may be used to produce photon-added states for a large class of signal-mode quantum states, such as thermal state, coherent state, and squeezed states[33]. Similarly, photon-subtracted states can be produced by means of conditional measurement on the BS[34]. Therefore, based on conditional measurement on the BS, it is possible to generate and manipulate various nonclassical optical fields in a real laboratory.

In this paper, we study the Hermite polynomial excited squeezed vacuum state (HESVS), a kind non-Gaussian quantum state, generated by conditional output mea-

surement on a BS. The calculations show that when a two-mode squeezed vacuum state (TMSVS) is injected in the input channels and the photon number of the mode in one of the output channels is measured, then the mode in the other output channel is prepared in a conditional state that has the typical features of a Hermite polynomial excited squeezed state. To exhibit the nonclassical properties of this conditional state, we mainly analyze the states in terms of the photon number distribution, sub-Poissonian distribution, quadrature component distribution, and Quasi-probability distribution including the Wigner function(WF) and Husimi function(HF). The paper is organized as follows. Section 2 presents the basic scheme for generation of the HESVS and its normalization related to Legendre polynomial. The nonclassical properties of the HESVS are analytically and numerically studied in Section 3-4. The results indicate that the conditional HESVS is strongly nonclassical and non-Gaussian due to the presence of the partial negative WF. Finally, a summary and concluding remarks are given in Section 5.

II. GENERATION OF HERMITE POLYNOMIAL EXCITED SQUEEZED STATE

It is well known that the input-output relations at a lossless beam splitter can be characterized by the SU(2) Lie algebra. In the Schrödinger picture, the role played by the beam splitter (BS) upon the input state ρ_{in} results in the output state

$$\rho_{out} = \hat{B}\rho_{in}\hat{B}^\dagger, \quad (1)$$

where $\hat{B} = \exp[\theta(a^\dagger b - ab^\dagger)]$ corresponds to the unitary operator in terms of the creation (annihilation) operator $a^\dagger(a)$ and $b^\dagger(b)$ for mode a and b , whose transformations satisfy[32]

$$\begin{aligned} \hat{B}a\hat{B}^\dagger &= a \cos \theta - b \sin \theta, \\ \hat{B}b\hat{B}^\dagger &= a \sin \theta + b \cos \theta. \end{aligned} \quad (2)$$

Moreover, $\cos \theta$ and $\sin \theta$ are the transmittance and reflectance of the beam splitter, respectively. Note that the globe phase factor of BS may be omitted without loss of generality. For the sake of simplicity, we also assume that θ is tunable in the range of $[0, \pi/2]$. Under special circumstances, when $\theta = 0$ or $\theta = \pi/2$, the BS corresponds to the cases of total transmission and total reflection, respectively. For $\theta = \pi/4$, the BS is just the symmetrical, i.e. 50/50 BS.

A. Hermite polynomial excited squeezed state

A two-mode squeezed vacuum state (TMSVS) is the correlated state of two field modes a and b (signal and idle) that can be generated by a nonlinear medium. Theoretically, the TMSVS is obtained by applying the unitary

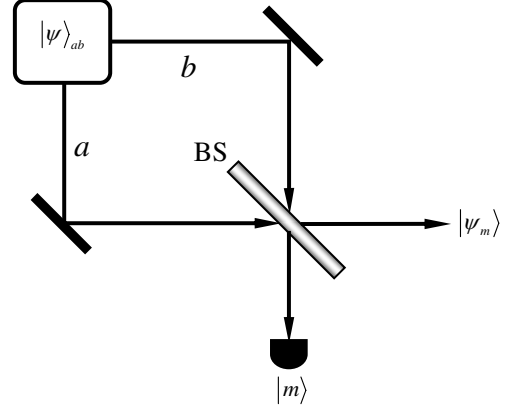


FIG. 1: Preparation scheme of HPESVS. When a TMSVS is mixed by a beam splitter and the number of photons $|m\rangle$ is measured in one of the output channels, then the conditional quantum state in the other output channel is generated.

operator $S_2(r)$ on the two-mode vacuum,

$$|\Psi\rangle_{ab} = S_2(r) |0, 0\rangle = \cosh^{-1} r e^{a^\dagger b^\dagger \tanh r} |0, 0\rangle, \quad (3)$$

where $S_2(r) = \exp[r(a^\dagger b^\dagger - ab)]$ is the two-mode squeezed operator and the values of r determines the degree of squeezing. The larger r , the more the state is squeezed. Especially, when $r = 0$, $|\Psi\rangle_{ab}$ reduces to two-mode vacuum state $|0, 0\rangle$.

The conceptual scheme of the experimental setup is depicted in Fig.1. The two input modes prepared in the two-mode squeezed state ($\rho_{in} = |\Psi\rangle_{ab} \langle\Psi|$) is mixed at BS, so the output-state density operator can be given by $\rho_{out} = \hat{B}|\Psi\rangle_{ab} \langle\Psi| \hat{B}^\dagger$. In fact the output modes in ρ_{out} are generally highly correlated. When the photon number of the mode in the second output channel is measured and m photons are detected, then the mode in the first output channel is prepared in a quantum state, whose density operator ρ_{out}^a reads as

$$\rho_{out}^a = N_m^{-1} {}_b \langle m | \hat{B} |\Psi\rangle_{ab} \langle\Psi| \hat{B}^\dagger |m\rangle_b = |\Psi_m\rangle \langle\Psi_m|, \quad (4)$$

where $|\Psi_m\rangle$ is the normalized output conditional state (a pure state) and N_m is the normalization factor determined by $\text{Tr}(\rho_{out}^a) = 1$.

Next, using the integration of the TMSVS[35],

$$|\Psi\rangle_{ab} = \frac{1}{\sinh r} \int \frac{d^2\alpha}{\pi} e^{-|\alpha|^2 / \tanh r + \alpha a^\dagger + \alpha^* b^\dagger} |0_a, 0_b\rangle, \quad (5)$$

where $|0_a, 0_b\rangle = |0\rangle_a \otimes |0\rangle_b$ is two-mode vacuum state, and the transformation relation in Eq.(2), after some al-

gebra we derive that

$$\begin{aligned}
& {}_b \langle m | \hat{B} | \Psi \rangle_{ab} \\
&= \frac{1}{\sinh r} \int \frac{d^2 \alpha}{\pi} e^{-|\alpha|^2 / \tanh r} e^{(\alpha \cos \theta + \alpha^* \sin \theta) a^\dagger} |0\rangle_a \\
&\times \frac{1}{\sqrt{m!}} \frac{\partial^m}{\partial \tau^m} e^{\tau(\alpha^* \cos \theta - \alpha \sin \theta)} \Big|_{\tau=0} \\
&= \frac{1}{\cosh r \sqrt{m!}} \frac{\partial^m}{\partial \tau^m} e^{\frac{\mu}{2} a^{\dagger 2} - \frac{\mu}{2} \tau^2 + \tau \nu a^\dagger} |0\rangle_a \Big|_{\tau=0}, \quad (6)
\end{aligned}$$

where $\mu = \sin 2\theta \tanh r$ and $\nu = \cos 2\theta \tanh r$. Hence the output conditional state $|\Psi_m\rangle$ is explicitly expressed as

$$|\Psi_m\rangle = \Omega_m^{1/2} H_m \left(\frac{\nu a^\dagger}{\sqrt{2\mu}} \right) S_1(\lambda) |0\rangle \quad (7)$$

with $\Omega_m = \mu^m \cosh \lambda / (2^m m! N_m \cosh^2 r)$, where we have used the generating function of the single-variable m -order Hermite polynomial $H_m(x) = \partial_\tau^m e^{2x\tau - \tau^2} \Big|_{\tau=0}$ and the expression of single-mode squeezed vacuum $S_1(\lambda) |0\rangle = \cosh^{-1/2} \lambda e^{(\tanh \lambda / 2) a^{\dagger 2}} |0\rangle$ with the single-mode squeezed operator $S_1(\lambda) = \exp[\lambda(a^{\dagger 2} - a^2)/2]$. Eq.(7) indicates that the conditional state $|\Psi_m\rangle$ is actually a single-mode m -order Hermite polynomial excited squeezed vacuum state. It is worth noticing that the degree of squeezing λ of the conditional state is not the same squeezing parameter r of the TMSVS but related to the parameters of the TMSVS and the BS satisfying $\tanh \lambda = \sin 2\theta \tanh r$. For the symmetrical case, i.e., $\theta = \pi/4$, $\hat{B} |\Psi\rangle_{ab} |_{\theta=\pi/4} = S_{1a}(r) |0\rangle \otimes S_{1b}(-r) |0\rangle$ is just the product state of two separate single-mode squeezed vacuum state. In this case, when we detect m photons in the second output channel, the conditional state is always $S_1(r) |0\rangle$ with the same squeezed parameter r . This is that the effect of the symmetrical BS splits the entangled TMSVS into two independent single-mode squeezed vacuum state. Note that when no photons are detected, $m = 0$, then $|\Psi_m\rangle$ also reduces to $S_1(\lambda) |0\rangle$.

B. Normalization via probability of such event

In addition, the normalization factor N_m is determined by the probability $p(m)$ of such an event given by

$$\begin{aligned}
N_m &= p(m) \\
&= \text{Tr} \left({}_b \langle m | \hat{B} | \Psi \rangle_{ab} \langle \Psi | \hat{B}^\dagger | m \rangle_b \right) \\
&= \frac{1}{m! \sqrt{A} \cosh^2 r} \frac{\partial^{2m}}{\partial s^m \partial \tau^m} e^{-B_1 s^2 / 2 - B_1 \tau^2 / 2 + B_2 s \tau} \Big|_{s=\tau=0} \\
&= \frac{(-\sqrt{B_3})^m}{\cosh^2 r \sqrt{A}} P_m(\sqrt{B_4}) \quad (8)
\end{aligned}$$

where we have set $A = 1 - \mu^2$, $B_1 = \mu / (A \cosh^2 r)$, $B_2 = \nu^2 / A$, $B_3 = (\tanh^4 r - \mu^2) / A$, $B_4 = \nu^4 / (A^2 B_3)$,

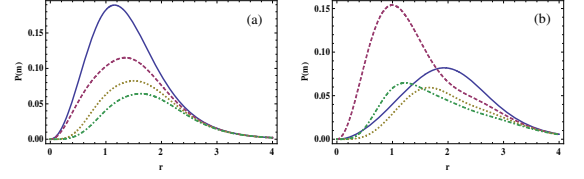


FIG. 2: The probability of producing $|\Psi_m\rangle$ is shown as a function of the parameter r of the TMSVS for two parameter values of the BS [(a) $\theta = \pi/7$; (b) $\theta = 2\pi/7$] and various values of m , where $m = 1, 2, 3, 4$ correspond to the solid, dashed, dotted and dotted-dashed lines, respectively.

and in the last step we have used the formula of m -order Legendre polynomial $P_m(x)$, i.e.,

$$\frac{\partial^{2m}}{\partial t^m \partial \tau^m} e^{-t^2 - \tau^2 + \frac{2x}{\sqrt{x^2 - 1}} \tau t} \Big|_{t, \tau=0} = \frac{2^m m!}{(x^2 - 1)^{m/2}} P_m(x). \quad (9)$$

Especially, when $r = 0$, $|\Psi\rangle \rightarrow |0, 0\rangle$, there is no photons in the output channels. In this case, there is no necessary to making conditional measurement. So the event is happen only for $r \neq 0$. When $\theta = 0$ or $\theta = \pi/2$, leading to $\mu = 0$ and $\nu = \tanh r$ or $\nu = -\tanh r$ then $A = 1$, $B_1 = 0$, $B_2 = \tanh^2 r$, $B_3 = \tanh^4 r$, and $B_4 = 1$, $N_m |_{\theta=0 \text{ or } \theta=\pi/2} = \tanh^{2m} r / \cosh^2 r$ and $|\Psi_m\rangle$ is just the Fock $|m\rangle$, which is rational because of the inherent properties of the TMSVS. If $\theta = \pi/4$, the BS is just the symmetrical, i.e., 50/50 BS, leading to $\mu = \tanh r$, $\nu = 0$ then $A = 1 - \tanh^2 r$, $B_1 = -\frac{\tanh^2 r}{2}$, $B_2 = 0$, then the output states is the the product of two independent single-mode SVS.

According to Eq.(8), we discuss the probability of observing such a conditional m -order HPESVS. In Fig.2 the probability $p(m)$ is plotted for two parameter values of the BS. For a given transmittance of BS, $p(m)$ as a function of the input squeezing parameter r can attain a maximum and the maximum is shifted towards larger values of r when m is increased (see Fig.2a). In this figure, for each r , we should use the transmittance of the BS in a way to optimize the success probability. The ideal procedure would use the appropriate transmittance for each value of r and m . By tuning the parameters of the interaction, namely, the control parameter of the BS, the squeezed parameter r of the TMSVS, and the photon number of conditional measurement m , the HPESVS may be modulated, generating a wide range of nonclassical phenomena, as described below.

III. OBSERVABLE NONCLASSICAL EFFECTS OF THE CONDITIONAL HPESVS

To study the nonclassical properties of the conditional states in more detail, we shall calculate the photon number distribution, sub-Poissonian distribution, and quadrature component distribution.

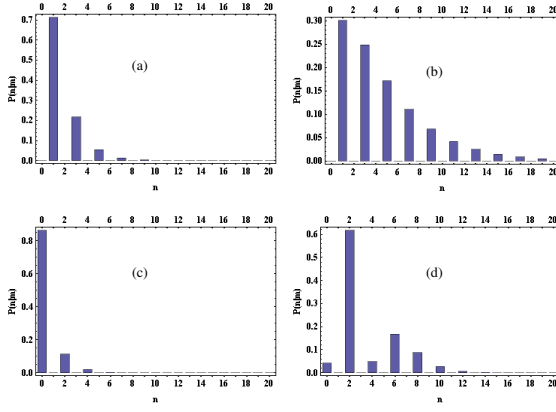


FIG. 3: Photon-number distribution of the conditional HPESVS for (a) $m = 1, \theta = 2\pi/7, r = 0.5$; (b) $m = 1, \theta = 2\pi/7, r = 1.0$; (c) $m = 4, \theta = 2\pi/7, r = 1.0$; (d) $m = 4, \theta = 3\pi/7, r = 1.0$, respectively.

A. Photon number distribution

The photon number distribution (PND), the probability of finding n photons, is a key characteristic of every quantum state. Recalling Eq.(7), the PND of the conditional HPESVS reads as

$$P(n|m) = |\langle n | \Psi_m \rangle|^2. \quad (10)$$

Using the unnormalized coherent $|z\rangle = \exp[za^\dagger]|0\rangle$, leading to $|n\rangle = \frac{1}{\sqrt{n!}} \frac{\partial^n}{\partial z^n} |z\rangle|_{z=0}$, and combining with Eq.(7), we finally obtain

$$\begin{aligned} P(n|m) &= \frac{1}{m!n!N_m \cosh^2 r} \left| \frac{\partial^{2m}}{\partial \tau^m \partial s^m} e^{\frac{\mu}{2}s^2 - \frac{\mu}{2}\tau^2 + \nu s\tau} \Big|_{s=\tau=0} \right|^2 \\ &= \frac{m!n!}{N_m \cosh^2 r} \left| \sum_{g=0}^{\min[m,n]} \frac{(-1)^{\frac{m-g}{2}} \left(\frac{\mu}{2}\right)^{\frac{m+n-2g}{2}} \nu^g}{\left(\frac{n-g}{2}\right)! \left(\frac{m-g}{2}\right)! g!} \right|^2, \quad (11) \end{aligned}$$

where we have used $\frac{\partial^m}{\partial x^m} x^n|_{x=0} = m! \delta_{mn}$ (δ_{mn} is Krocher function) and N_m is given in Eq.(8). In the summation of Eq.(11), the value of g must make $\frac{n-g}{2}$ and $\frac{m-g}{2}$ be integer. To see clearly the variation of the PND, in Fig.3 we plot the bar graph of the PND for the conditional HPESVS with different values of parameters m , θ , and r . From Fig.3 we easily see that when the number m of the conditional measurement is odd (even), then the photon-number distribution is nonzero only for odd (even) photon numbers. The probability $P(n|m)$ with different parity between m and n is zero. For given m and θ , the bigger the squeezing parameter r , the wider the distribution (see Figs.3(a) and 3(b)).

B. Sub-Poissonian distribution

In order to study the photon-number statistics of this conditional state, we first calculate $\langle a^k a^{\dagger l} \rangle = \langle \Psi_m | a^k a^{\dagger l} | \Psi_m \rangle$. Using the completeness of coherent state $\int \frac{d^2\alpha}{\pi} |\alpha\rangle \langle \alpha| = 1$ as well as $y^k = \frac{\partial^k}{\partial t^k} e^{ty} |_{t=0}$ yields

$$\begin{aligned} \langle a^k a^{\dagger l} \rangle &= \frac{1}{N_m m! \cosh^2 r \sqrt{A}} \frac{\partial^{2m}}{\partial s^m \partial \tau^m} e^{-B_1 s^2/2 - B_1 \tau^2/2 + B_2 \tau s} \\ &\times \frac{\partial^{k+l}}{\partial x^k \partial y^l} e^{[\mu\nu(xs+y\tau) + \nu(ys+x\tau) + \mu(x^2+y^2)/2 + xy]/A} \Big|_{s=\tau=x=y=0}. \quad (12) \end{aligned}$$

Thus, the mean photon number

$$\langle n \rangle = \langle a^\dagger a \rangle = \langle aa^\dagger \rangle - 1, \quad (13)$$

can be determined by Eq.(12) with $k = l = 1$. Examples are shown in Figs.4(a) and 4(c). We see that when $\theta = \pi/5$, the number of photons that can be found in $|\Psi_m\rangle$ increases with m for the given larger r , and $\langle n \rangle$ as a function of θ is symmetric distribution for $\theta = \pi/4$. This is simply a consequence of the BS transformation. In particular, when no photons are detected, $m = 0$, then $\langle n \rangle = \sinh^2 r$ reduces to the mean photon number of single-mode squeezed vacuum state.

A measure of the deviation of the photon number distribution from a Poissonian is the Mandel Q factor defined by[36]

$$\begin{aligned} Q &= \frac{\langle n^2 \rangle - \langle n \rangle^2}{\langle n \rangle} - 1 \\ &= \frac{\langle a^2 a^{\dagger 2} \rangle - \langle aa^\dagger \rangle^2 - 2\langle aa^\dagger \rangle + 1}{\langle aa^\dagger \rangle - 1}, \quad (14) \end{aligned}$$

It holds that $Q \geq 0$ and the equality is achieved for the Fock state. The light is sub-Poissonian when the photon-number variance $\langle n^2 \rangle - \langle n \rangle^2$ is less than $\langle n \rangle$. This is indicated by a negative value of Q . The statistics are Poissonian when $Q = 0$, and super- (sub-) Poissonian if $Q > 0$ ($Q < 0$).

According to Eqs.(12) and (14), we plot the variation of Q for HPESVS versus r or θ for different $m = 1, 2, 3, 4$ in Fig.4(b) and 4(d). It is clearly seen that Q as a function of θ is also symmetric distribution for $\theta = \pi/4$ and the HPESVS has sub-Poissonian statistics behavior due to the emergence of the negativity of Q . With the increasing values of m , the increasing trend of Q is accelerated for larger r . To further exhibit the high nonclassicality, Fig.5 shows the dependence of this conditional state on θ and r for four different Q factors. Especially, we consider first the boundary case of the Poissonian distribution, $Q = 0$ (see the solid line in Fig.5).

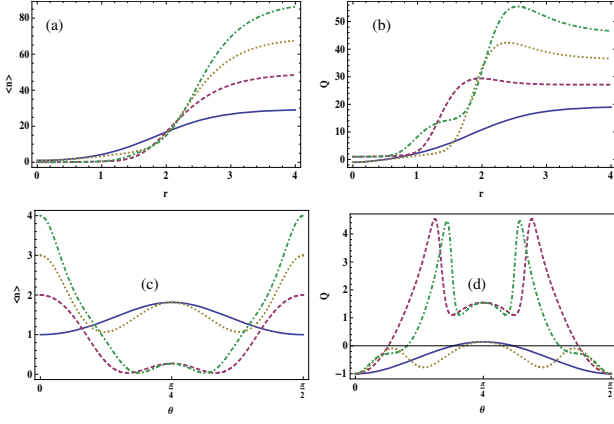


FIG. 4: (a) Mean photon number $\langle n \rangle$ and (b) Mandel Q factor versus r for different $m = 1$ (solid line), $m = 2$ (dashed line), $m = 3$ (dotted line), and $m = 4$ (dotdashed line) with the same $\theta = \pi/5$. (c) Mean photon number $\langle n \rangle$ and (d) Mandel Q factor versus θ for different $m = 1$ (solid line), $m = 2$ (dashed line), $m = 3$ (dotted line), and $m = 4$ (dotdashed line) with the same $r = 0.5$.

C. Quadrature component distribution

Next we pay attention to the conditional quadrature component distribution (QCD)[30]

$$P(x, \varphi | m) = |\langle x, \varphi | \Psi_m \rangle|^2, \quad (15)$$

which can be measured in balanced homodyne detection. Here $|x, \varphi\rangle$ is the eigenstate of the quadrature component $X(\varphi) = (ae^{-i\varphi} + a^\dagger e^{i\varphi})$, expressed as in the Fock basis

$$|x, \varphi\rangle = \pi^{-1/4} e^{-\frac{x^2}{2} + \sqrt{2}xa^\dagger e^{i\varphi} - \frac{a^{\dagger 2}e^{2i\varphi}}{2}} |0\rangle. \quad (16)$$

Using Eqs.(7) and (16) and inserting the completeness of coherent state $\int \frac{d^2\alpha}{\pi} |\alpha\rangle \langle \alpha| = 1$, after integration, the wave function $\langle x, \varphi | \Psi_m \rangle$ reads

$$\langle x, \varphi | \Psi_m \rangle = \frac{\pi^{-1/4} \left(\sqrt{\Gamma/2} \right)^m e^{-\frac{\Pi}{2}x^2}}{\sqrt{N_m m!} (1 + \mu e^{-2i\varphi}) \cosh r} H_m \left(\frac{\Delta}{\sqrt{\Gamma}} x \right), \quad (17)$$

where we have set $\Theta = 1 + \mu e^{-2i\varphi}$, $\Pi = (1 - \mu e^{-2i\varphi}) / \Theta$, $\Gamma = (\mu + e^{-2i\varphi} \tanh^2 r) / \Theta$, and $\Delta = e^{-i\varphi} \nu / \Theta$.

As a result of Eq.(17), we easily obtain the conditional QCD defined by Eq.(15) and plot the variation of $P(x, \varphi | m)$ for the HPESVS as a function of x or φ for different $m = 1, 2, 3, 4$ in Fig.6. One sees that for φ near $\pi/2$ the QCD $P(x, \varphi | m)$ with $m > 0$ exhibits two separated peaks, whereas for φ close to 0 and π an interference pattern is observed.

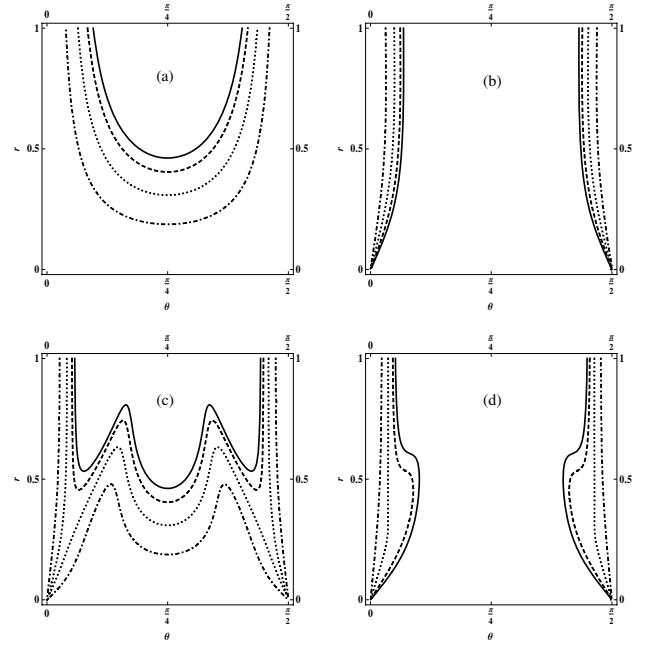


FIG. 5: The sub-Poissonian properties of the conditional state with $Q = 0$ (solid line), $Q = -0.2$ (dashed line), $Q = -0.5$ (dotted line), and $Q = -0.8$ (dotdashed line) in the plane space of two parameters (θ and r) with different number of conditional measurement: (a) $m = 1$; (b) $m = 2$; (c) $m = 3$; (d) $m = 4$, respectively.

IV. QUASI-PROBABILITY DISTRIBUTION OF THE CONDITIONAL HPESVS

Quasi-probability distribution function in the phase space is a very useful tool for a comprehensive description of the nonclassical state. Thus, in this section, we shall analytically discuss several quasi-probability distributions, including Wigner function and Husimi function to characterize the nonclassicality of the conditional HPESVS.

A. Wigner function

The WF was first introduced by Wigner in 1932 to calculate quantum correction to a classical distribution function of a quantum-mechanical system. The presence of negativity of the WF is a signature of its nonclassicality[38, 39]. For a single-mode density operator ρ , the WF in the coherent state representation $|z\rangle$ can be expressed as

$$W(\alpha) = \frac{2e^{2|\alpha|^2}}{\pi} \int \frac{d^2z}{\pi} \langle -z | \rho | z \rangle e^{-2(z\alpha^* - z^*\alpha)}, \quad (18)$$

where $\alpha = (x + ip)/\sqrt{2}$. The Wigner function of the conditional state $\rho_{out}^a = |\Psi_m\rangle \langle \Psi_m|$, can be calculated in

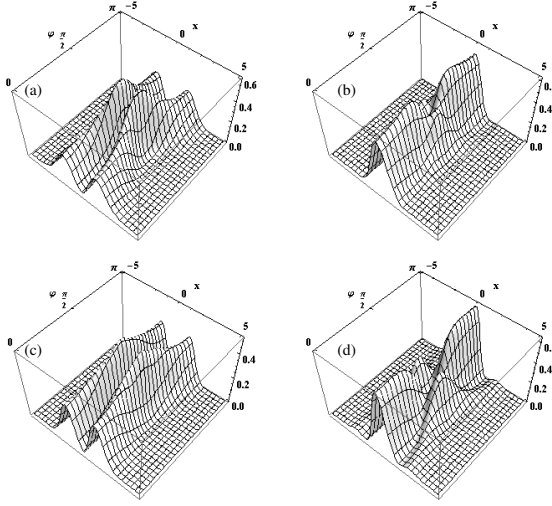


FIG. 6: Quadrature-component distribution $P(x, \varphi|m)$ of the conditional state $|\Psi_m\rangle$ for $\theta = \pi/7, r = 0.5$ and various numbers m of measured photons with (a) $m = 1$, (b) $m = 2$, (c) $m = 3$, and (d) $m = 4$, respectively.

a straightforward way.

$$\begin{aligned}
 W(x, p|m) &= \frac{2}{\pi N_m m! \cosh^2 r \sqrt{A}} e^{-2\Xi|\alpha|^2 + \frac{2\mu}{A}\alpha^2 + \frac{2\mu}{A}\alpha^{*2}} \\
 &\times \frac{\partial^{2m}}{\partial \tau^m \partial s^m} e^{Rs + R^* \tau - B_2 \tau s - \frac{B_1}{2} s^2 - \frac{B_1}{2} \tau^2} \Big|_{s=\tau=0} \\
 &= \frac{2m!}{\pi N_m \cosh^2 r \sqrt{A}} e^{-2\Xi|\alpha|^2 + \frac{2\mu}{A}\alpha^2 + \frac{2\mu}{A}\alpha^{*2}} \\
 &\times \sum_{l=0}^m \frac{(-B_2)^l (-B_1/2)^{m-l}}{l! [(m-l)!]^2} \left| H_{m-l} \left(-\frac{R}{\sqrt{2B_1}} \right) \right|^2, \quad (19)
 \end{aligned}$$

where we have set $\Xi = (1 + \mu^2) / (1 - \mu^2)$ and $R = 2\nu(\alpha - \mu\alpha^*)/A$. Especially, when no photons are detected, $m = 0$, then $W(x, p|0) \rightarrow \exp(-p^2 e^{-2\lambda} - x^2 e^{2\lambda})/\pi$ is a Gaussian form in phase space, which is just WF of single-mode SVS, as expected.

The WFs of the conditional HPESVS $|\Psi_m\rangle$ in Fig.7 are plotted for the same parameters as in Fig.6. The figures indicate that the conditional HPESVS is a nonclassical non-Gaussian state, since the partial negative regions in phase space are observed in Fig.7. This further demonstrates that performing the conditional output measurement on a BS is an effective approach to generate non-Gaussian state. In addition, it is seen from Fig.7 that for odd m there exists a negative valley in the center region, whereas for even m there exists a main peak. In fact, for the center region $W(0, 0|m) = \frac{2}{\pi} (-1)^m$, as expected.

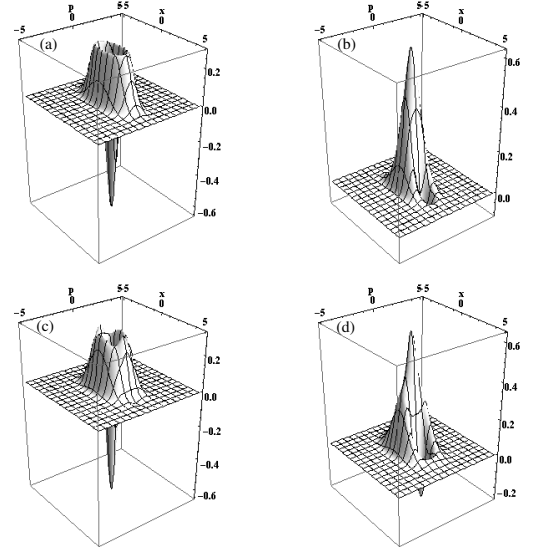


FIG. 7: Wigner functions $W(x, p|m)$ of the conditional state $|\Psi_m\rangle$ for $\theta = \pi/7, r = 0.5$ and various numbers m of measured photons with (a) $m = 1$, (b) $m = 2$, (c) $m = 3$, (d) $m = 4$, respectively.

B. Husimi function

The Husimi function $Q(x, p|m)$ of the state $|\Psi_m\rangle$ is defined by[29]

$$Q(x, p|m) = \frac{1}{\pi} |\langle \beta | \Psi_m \rangle|^2, \quad (20)$$

where $|\beta\rangle$ is a coherent state and $\beta = (x + ip)/\sqrt{2}$. Using Eq.(7), the scalar product $\langle \beta | \Psi_m \rangle$ can be easily calculated as follow

$$\langle \beta | \Psi_m \rangle = \frac{e^{-\frac{|\beta|^2}{2} + \frac{\mu}{2}\beta^{*2}}}{\sqrt{m!N_m} \cosh r} \frac{\partial^m}{\partial \tau^m} e^{-\tau^2 \frac{\mu}{2} + \tau \nu \beta^*} \Big|_{\tau=0}, \quad (21)$$

So we find that $Q(x, p|m)$ can be written as

$$Q(x, p|m) = \frac{(\mu/2)^m e^{-|\beta|^2 + \frac{\mu}{2}(\beta^2 + \beta^{*2})}}{\pi N_m m! \cosh^2 r} \left| H_m \left(\frac{\nu}{\sqrt{2\mu}} \beta^* \right) \right|^2. \quad (22)$$

As expected, for $m = 0$ the Husimi function is Gaussian, while for odd m a two-peak structure and for even m a single peak are observed in Fig.8, whose parameters are the same as WFs in Fig.7. Note that the Husimi function is a phase-space function that can be measured in multiport balanced homodyning. Since the Husimi function can be regarded as a smoothed Wigner function, it is always non-negative and the oscillating behavior, typical of WF (see Fig.7), cannot be observed.

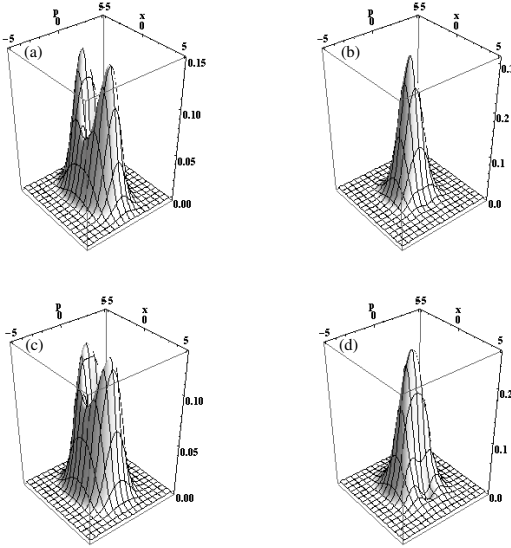


FIG. 8: Husimi functions $Q(x, p|m)$ of the conditional state $|\Psi_m\rangle$ for $\theta=\pi/7$, $r=0.5$ and various numbers m of measured photons with (a) $m=1$, (b) $m=2$, (c) $m=3$, (d) $m=4$, respectively.

V. CONCLUSIONS AND DISCUSSIONS

In summary, we have shown that Hermite polynomial excited squeezed states can be generated by con-

ditional measurements using a simple beam splitter scheme. When a two-mode squeezed vacuum state is mixed by a beam splitter and the number of photons is measured in one of the output channels, then the conditional quantum state in the other output channel reveals all properties of a Hermite polynomial excited squeezed state. Then, we also have numerically analyzed the conditional HPESVS in terms of the photon-number statistics, quadrature-component distribution and quasi-probability distribution such as the Wigner and Husimi functions. The results show that by tuning the parameters of the interaction, namely, the control parameter of the BS, the squeezed parameter r of the TMSVS, and the photon number of conditional measurement m , the HPESVS may be modulated, generating a wide range of nonclassical phenomena. This further demonstrates that performing the conditional measurement on a BS is an effective approach to generate non-Gaussian state.

Acknowledgments

This project was supported by the National Nature Science Foundation of China (Nos.11264018 and 11447002).

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